Continuous Headtail for Measuring Chromaticity

C.Y. Tan
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What is Continuous Headtail?

- Small continuous transverse kicks near the betatron tune.
- The head and tail of the bunch oscillate at a different phase
  - The constant phase difference is proportional to chromaticity.
- Contrast this with normal headtail technique
  - Single kick
  - Head and tail oscillates with different phase. The maximum phase difference is used to calculate chromaticity

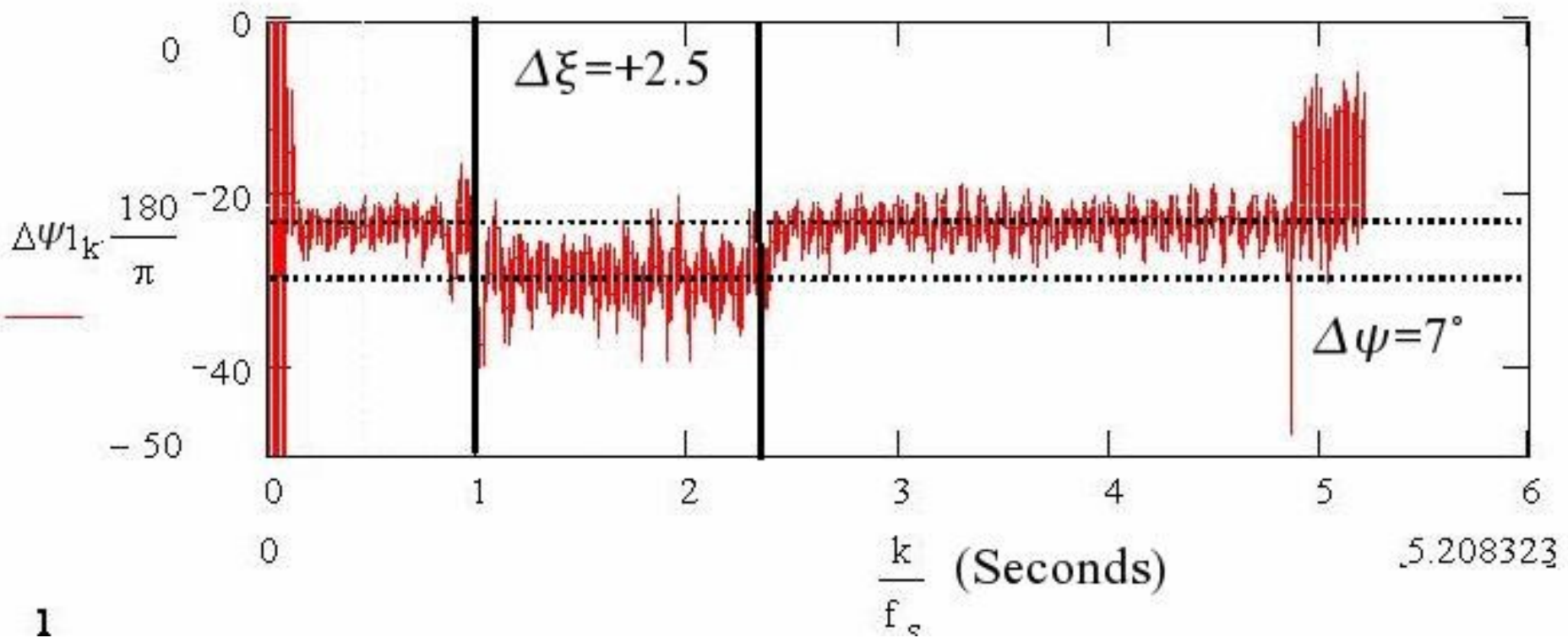
Advantages

- Compatible with tune tracker
  - Tune tracker already kicks the beam transversely and close to the betatron tune.
- No extra modulations are required.
  - c.f. Traditional technique, McGinnis method etc.
- Can use 3D-BBQ to measure the phase of the head and tail separately.

FNAL Task

- To verify that the initial observations at BNL were indeed real.
- We volunteered to derive the equation which relates chromaticity to phase difference between head and tail *if this effect is real.*
- Perform an experiment to verify the calculations and simulations.

First Observation at RHIC



The phase difference between the head and tail with continuous kicks was measured by the M. Gasior in the SPS during the machine studies period on 29 Sep 2006. The data was analysed by V. Ranjbar. This showed that the head-tail phase changed when the chromaticity was increased by 2.5~units.

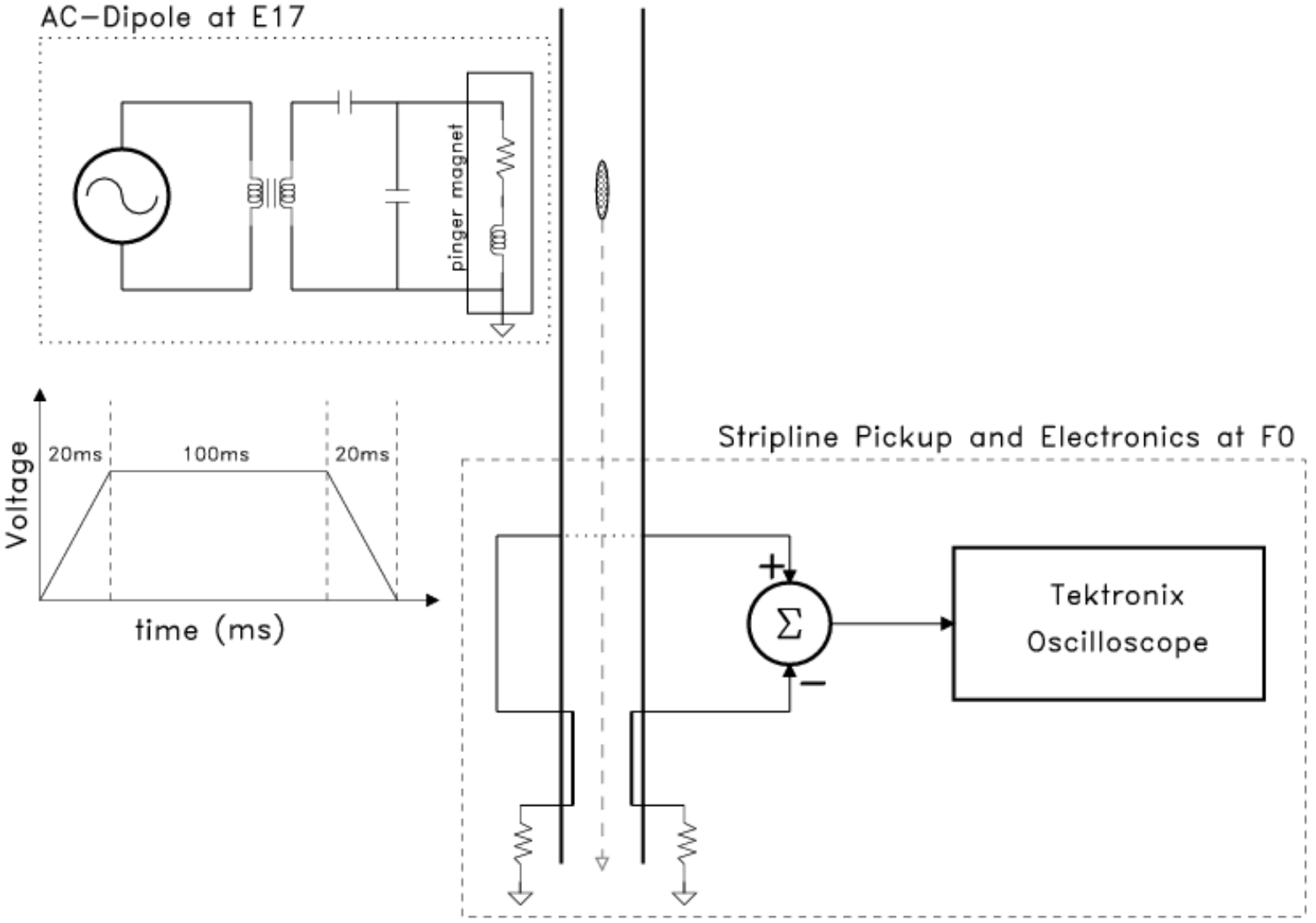
Is this real?

- This data was taken at SPS: there were only a few data sets out of 20 which showed this effect.
- There was an on going “bet” between CERN and FNAL about whether this could actually work.

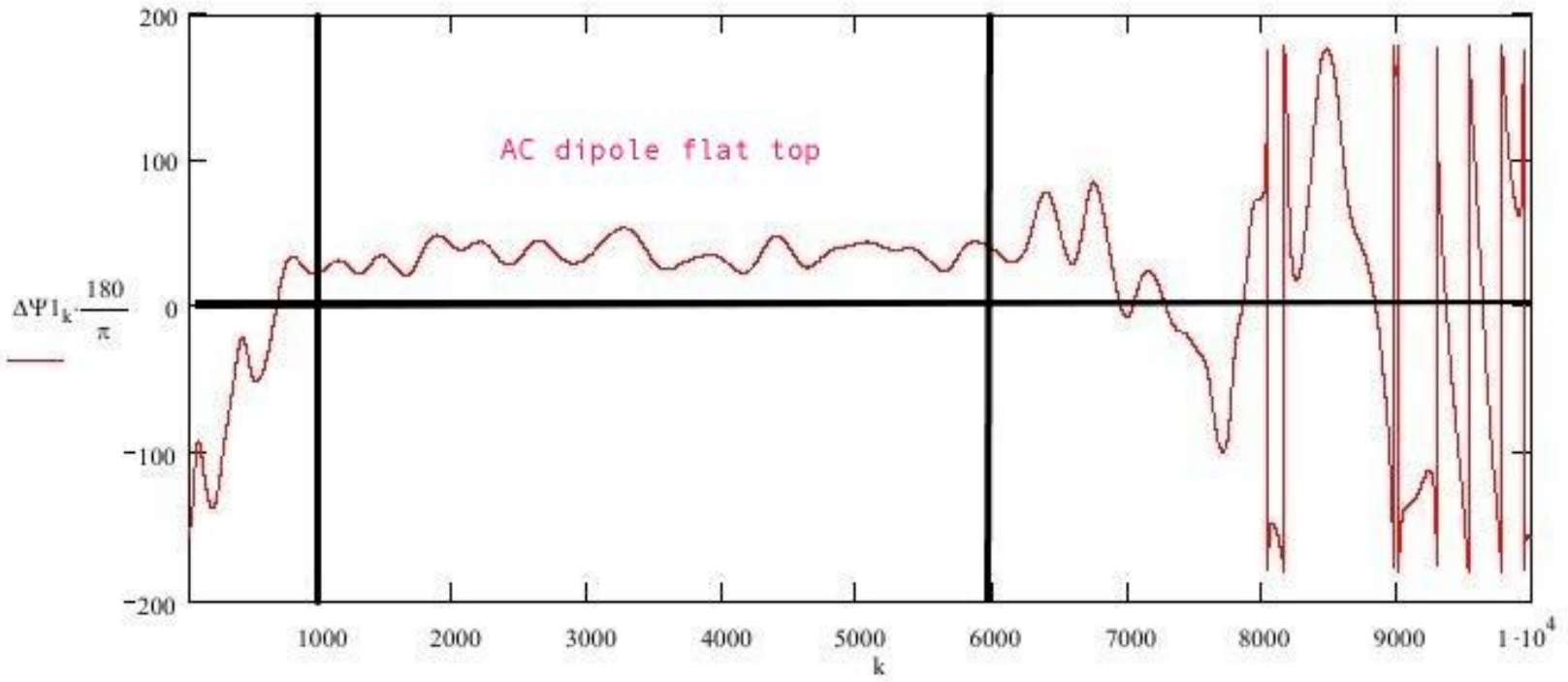
The Experiment

- We decided to use the AC-dipole to kick the beam continuously.
  - To divorce ourselves from the 3D-BBQ which may introduce unknown artefacts, plus we really don't know the exact position at the head/tail when we use it.
  - Large oscillations so that the signal will be unambiguous and thus yield good phase differences.

Experimental Setup



There is a phase difference!



So, we do measure a phase difference between the head and the tail! Lost bet with CERN, so have to derive formula

The Formula

Solve Hill's equation with external periodic forcing

$$\ddot{x}+\left[2\pi Q+2\pi\xi\left(\delta_0\cos(2\pi Q_s n)-\frac{\omega_s\tau_0}{\eta}\sin(2\pi Q_s n)\right)\right]^2 x=\epsilon\lambda\cos(2\pi Q_k n)$$

After many pages of maths (See TM-2376)

$$\Delta\psi=\frac{2\xi\omega_0\tau_B}{\eta}$$

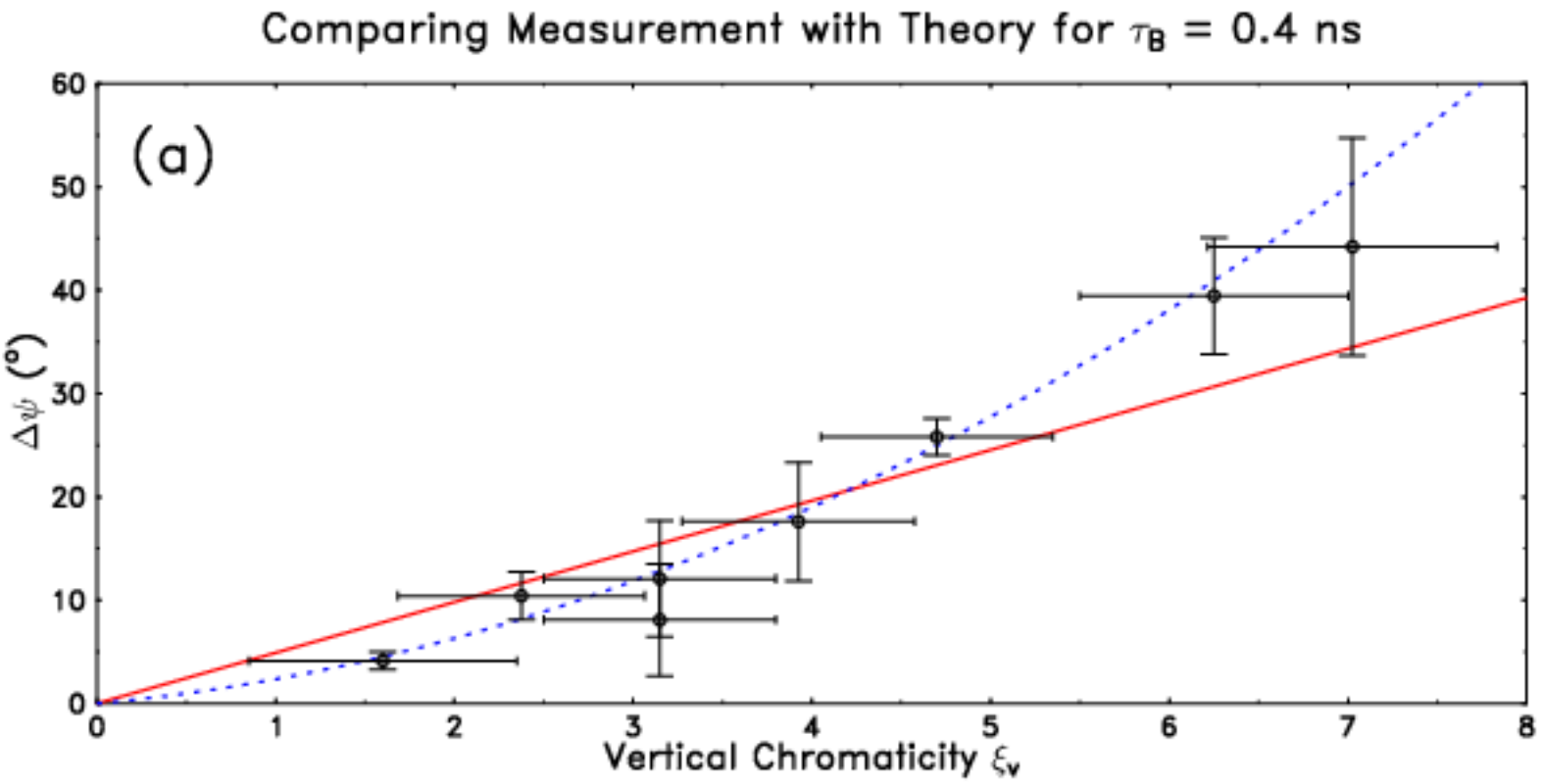
$\Delta\psi=$ phase of head w.r.t. tail $\quad\quad$ $\xi=$ chromaticity

$\omega_0=2\pi$ revolution frequency $\quad$ $\tau_B=$ distance in ns from centre of bunch

$\eta=$ slip factor

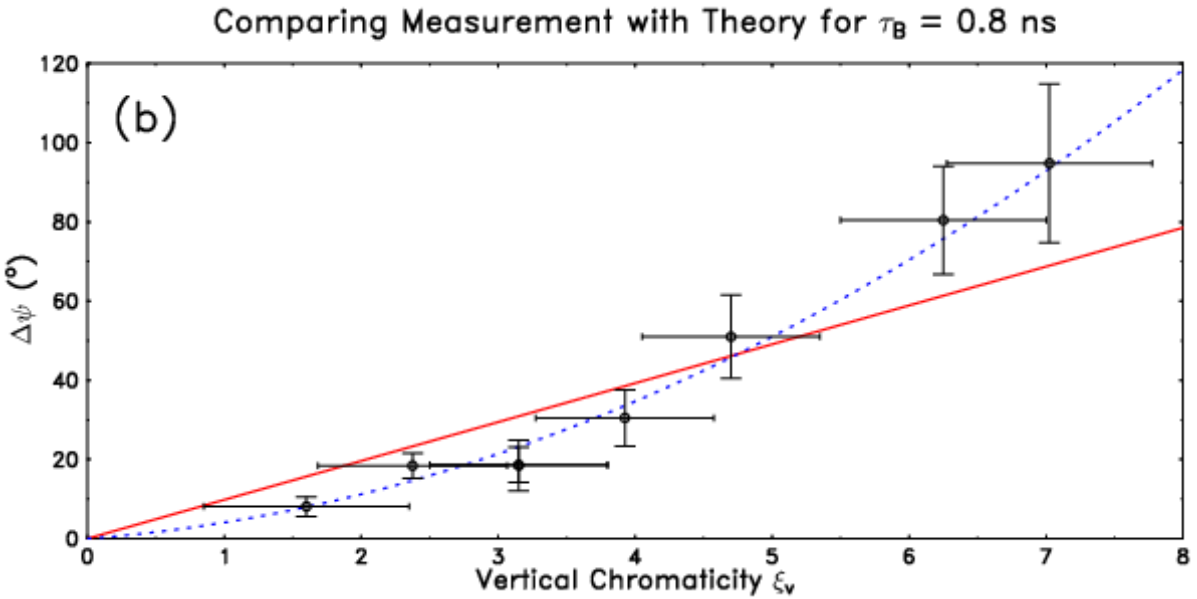
Formula is true for zero transverse emittance beam.
Same formula derived by Fartoukh using different method. At least verified by someone else.

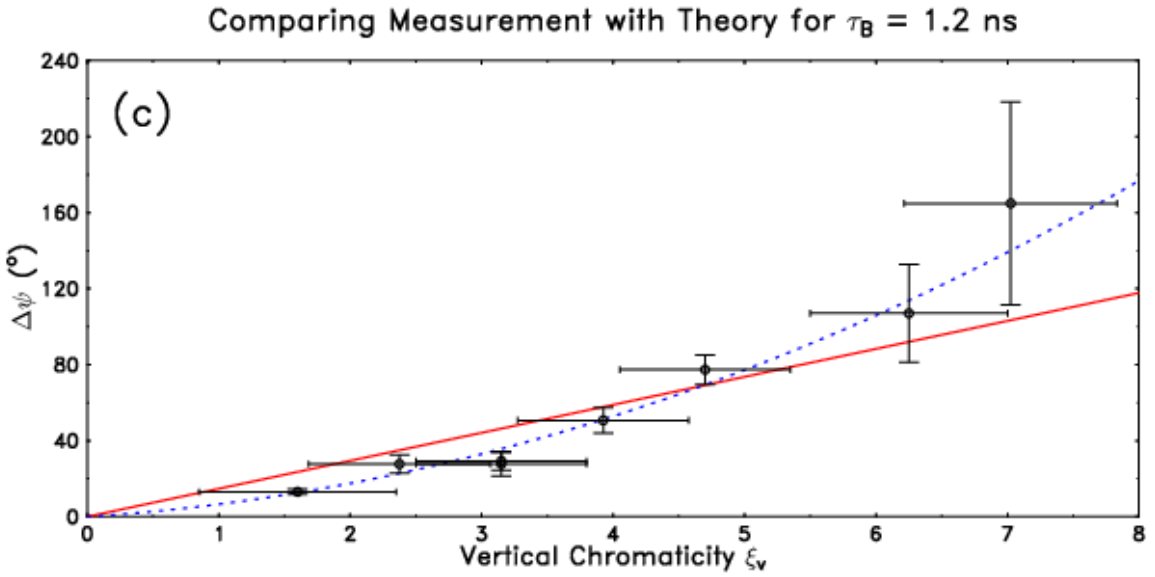
The Results



Red line is theory.
Blue dotted line is quadratic fit.
There is a clear quadratic component!
Bunch length σ = 2.9 ns

The Results (cont'd)

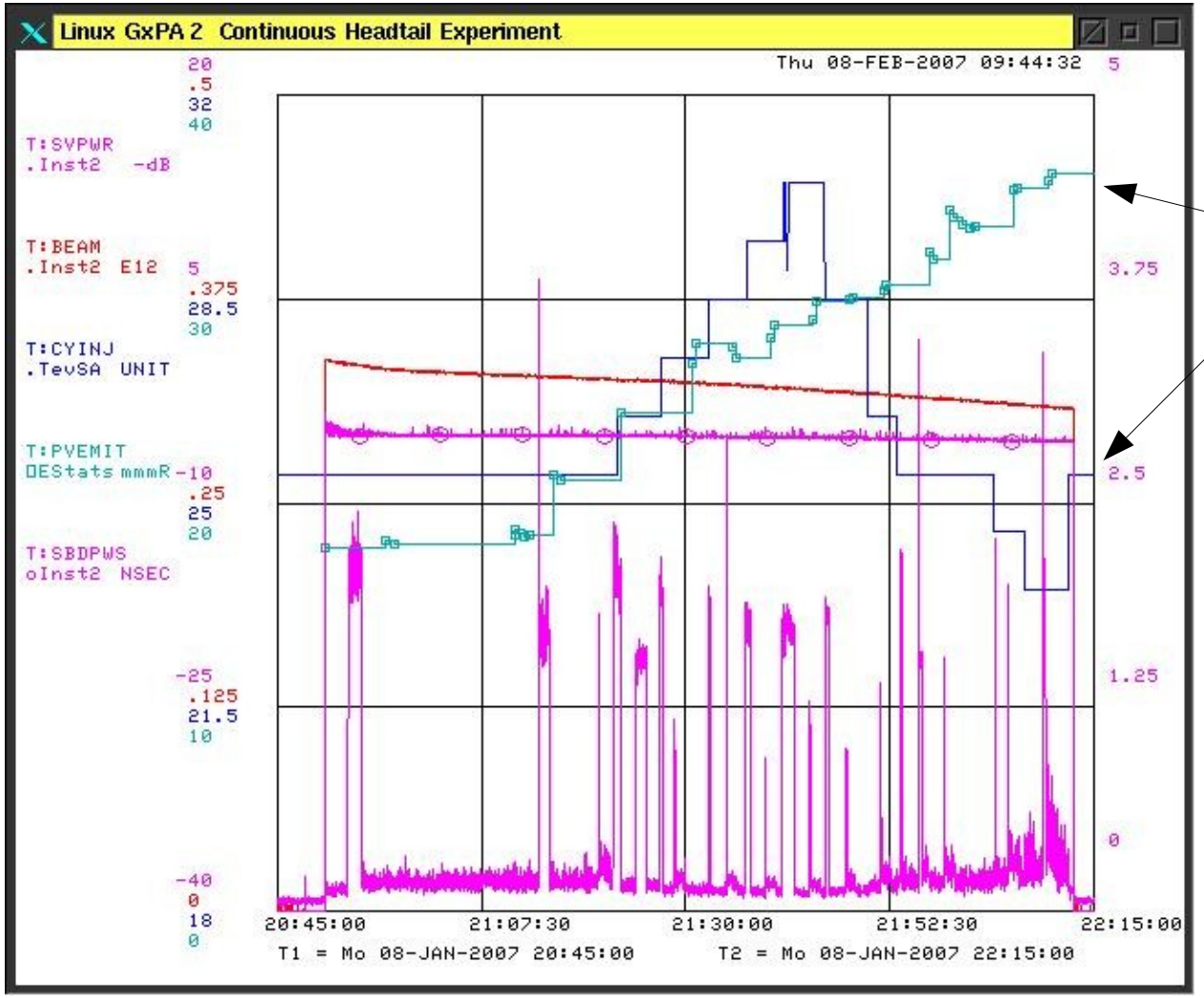




The Data Tells Us that ...

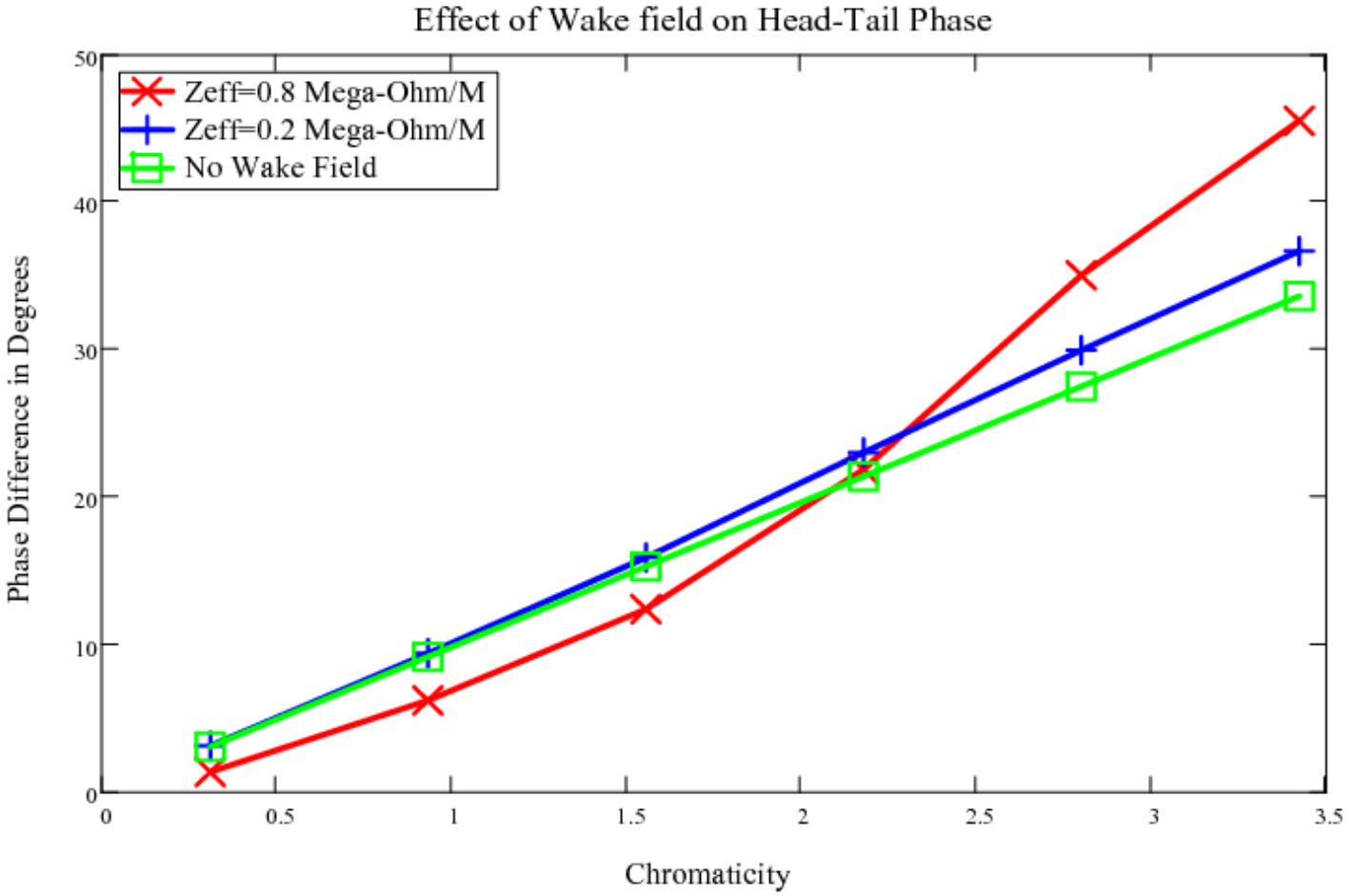
- There is an unambiguous phase difference between the head and the tail.
- The theory does not seem to fit the data.
- There is a quadratic term which needs to be resolved ...

So where does the quadratic term come from?



Emittance growing and non zero! Lower chroms measured when emittance is larger

Impedance?



Simulation done for SINGLE kick

Conclusion

- There is a relationship between phase difference and chromaticity.
  - Impedance is the likely candidate for quadratic term.
- Must use 3D-BBQ to see if results are reproducible.